Modified-Strip-Analysis Method for Predicting Wing Flutter at Subsonic to Hypersonic Speeds

E. Carson Yates Jr.*
NASA Langley Research Center, Hampton, Va.

The modified-strip-analysis method of flutter prediction, which was originally presented by the author in an earlier work, is reviewed briefly, some of its limitations are examined, its relations to some other strip methods are indicated, and some results of its use are shown in comparison with experimental flutter data and with results of other analytical methods of Mach numbers up to 15.3. The modified strip analysis is formulated from Theodorsen's method and employs distributions of aerodynamic parameters which may be evaluated from any suitable linear or nonlinear steady-flow theory or from measured steady-flow load distributions for the undeformed wing. The method has been shown to give good flutter results for a broad range of swept and unswept wings at speeds up to hypersonic. The method, however, is not suitable for application to wings of very low aspect ratio nor to unswept wings at Mach numbers near 1.0.

Nomenclature

A = aspect ratio of full wing including fuselage intercept
a = nondimensional distance from midchord to elastic axis measured perpendicular to elastic axis, positive rearward, fraction of semichord b
$a_{c,n}$ = nondimensional distance from midchord to local aerodynamic center (for steady flow) measured perpendicular to elastic axis, positive rearward, fraction of semichord b
b = semichord of wing measured perpendicular to elastic axis
b_s = streamwise semichord at wing root
$C = F + iG$ = complex circulation function
$c_{\ell_{\alpha,n}}$ = local lift-curve slope for a section perpendicular to elastic axis in steady flow
h = local vertical translational displacement of wing at elastic axis, positive downward
k = reduced frequency, $\omega b/V_n$
M = Mach number
M_α = oscillatory moment about elastic axis per unit length of wing, positive leading edge up
P = oscillatory lift per unit length of wing along elastic axis, positive downward
w = downwash expression defined by Eq. (6)
V = freestream speed
V_n = component of freestream velocity normal to elastic axis
θ = local torsional displacement of wing measured about elastic axis, positive leading edge up
$\Lambda_{c/4}$ = quarter-chord sweep angle, positive for sweepback
Λ_{ea} = elastic-axis sweep angle, positive for sweepback
λ = taper ratio of full wing including fuselage intercept
μ = mass ratio, mass of wing panel divided by mass of fluid at freestream density contained in a conical frustum having wing root and tip chords as base diameters and panel span as height
ρ = fluid density
σ = local bending slope of elastic axis
τ = local rate of change of twist along elastic axis
ω = flutter frequency
ω_α = frequency of first uncoupled torsional vibration mode

Subscripts

C = quantities associated with compressible flow
I = quantities associated with incompressible flow
$(\dot{\ })$ = dot over a quantity indicates differentiation with respect to time

Introduction

In subsonic and supersonic flutter analyses for finite wings, evaluation of the required oscillatory aerodynamic loads by rigorous methods usually entails extensive computation. Even with the aid of high-speed computing machinery, flutter analyses by some of the more involved methods are arduous and time-consuming. Some of these procedures, therefore, have not been widely used. Consequently, approximate methods, notably strip methods, are frequently employed for trend studies, for preliminary design work, and for examining the mechanism of flutter, whereas the more erudite methods are frequently reserved for checking final designs.

Strip-type adaptations of two-dimensional methods to three-dimensional flutter problems, moreover, often require extensive tabulations of loading parameters or rather complicated expressions programed for their generation. Exceptions to this statement are the strip-type application of Theodorsen's method[1] to unswept wings[2] and the adaptation of that method for swept wings by Barmby, Cunningham, and Garrick.[3] Although these methods are based on two-dimensional incompressible flow, they can be modified to take into account the aerodynamic effects of finite span and compressibility.

Reference 4 presented a simple, approximate, modified-strip-analysis method for rapid flutter prediction which was essentially a generalization of the method of Barmby, Cunningham, and Garrick in order to account for the aerodynamic effects of finite planform and compressibility. The purpose of the present report is to review this method briefly, to examine some of its limitations, to indicate its relation to some other strip methods, and to summarize some of the results obtained with it.

Description of Modified Strip Analysis

In the modified strip analysis, as presently formulated, the oscillatory aerodynamic loads are evaluated in terms of wing

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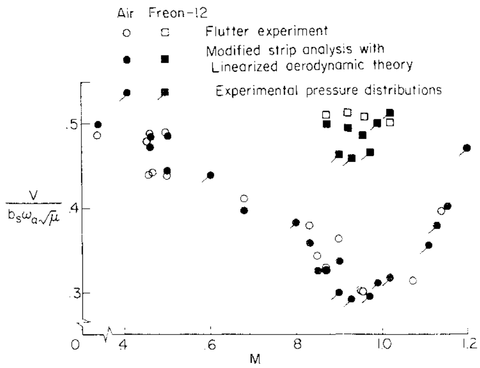
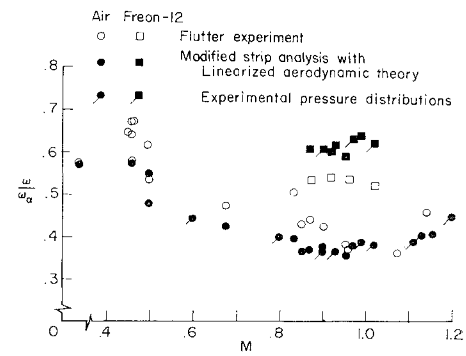
* Aerospace Engineer. Associate Fellow Member AIAA.



a) Flutter speeds.



b) Flutter frequencies.

Fig. 1 Flutter characteristics of swept wing in air and in Freon-12: $\Lambda_{c/4} = 45°$, $\lambda = 0.6$, and $A = 4.0$.

sections oriented normal to the wing elastic axis as in the method of Barmby, Cunningham, and Garrick,[3] although an analogous procedure would apply for streamwise sections. The fundamental concept underlying this modification of the method of Barmby, Cunningham, and Garrick may be stated as follows: For the oscillatory as well as for the steady-flow condition, the dominant aerodynamic effects of finite planform and compressibility are considered to be indicated by the lift-and pitching-moment-producing capacity of each wing section as reflected by the steady-state section lift-curve slope and aerodynamic-center position for the undeformed wing.

The nature of the present modification may be more clearly indicated by illustrating the relation of the modified strip analysis to the method of Barmby, Cunningham, and Garrick (or to the method of Theodorsen) and to the steady-state condition. For this purpose, consider the lift force acting on a streamwise section of a wing, which for simplicity is assumed to be unswept. For the steady-state condition in three-dimensional compressible flow

$$
\begin{aligned}
P &= -c_{\ell_{\alpha,n}}\theta(\rho/2)V^2(2b) \\
&= -c_{\ell_{\alpha,n}}\rho V b w
\end{aligned}
\tag{1}
$$

where w is the downwash such that the effective angle of attack is given by w/V. Theodorsen[1] effectively modified this expression (and the corresponding expression for pitching moment) for application to a thin wing oscillating in two-dimensional incompressible flow. For this case $c_{\ell_{\alpha,n}} = 2\pi$, and

$$P = -2\pi\rho V b w C + \text{noncirculatory terms} \tag{2}$$

where w is now the effective downwash associated with the oscillation, $C = C(k) = F_I + iG_I$ is a complex "circulation function," which accounts for the effect of the oscillatory motion on the magnitude and phase angle of the lift vector, and the noncirculatory terms account for the air forces associated with the up-and-down pumping action of the wing as distinguished from the circulatory or induced forces represented by the first term of Eq. (2). In formulating the corresponding pitching moment M_α, the section aerodynamic center is located at the quarter chord for two-dimensional incompressible flow. The method of Barmby, Cunningham, and Garrick is, in turn, a stripwise adaptation of Theodorsen's equations for application to swept wings. For unswept wings the two methods coincide.

In the analogous expressions for the modified strip analysis, the section lift-curve slope and aerodynamic center remain arbitrary and thus may vary from section to section across the span and may also vary with Mach number. Thus for oscillatory three-dimensional compressible flow

$$P = -c_{\ell_{\alpha,n}}\rho V b w C + \text{noncirculatory terms} \tag{3}$$

The values of section lift-curve slope and of section aerodynamic center may be evaluated from any suitable linear or nonlinear steady-flow aerodynamic theory,[4-10] or from measured load distributions,[8-11] in other words, from any method considered to give accurate steady-state loads. For nonzero Mach numbers the complex circulation function of Theordorsen is modified in magnitude only by utilizing aerodynamic coefficients given by Jordan[12] for a two-dimensional thin airfoil oscillating in subsonic or supersonic flow. Thus

$$C = C(k, M) = [(F_C^2 + G_C^2)/(F_I^2 + G_I^2)]^{1/2}(F_I + iG_I)$$

or for moderately small reduced frequencies

$$C \approx (F_I + iG_I)F_C/F_I$$

Here subscript I designates values associated with incompressible flow; whereas, subscript C denotes values calculated from Jordan's coefficients for compressible flow. The underlying idea here is that if the reduced frequency is moderately small, the G values remain relatively small, and the calculated flutter speed is not very sensitive to changes in G. Furthermore, the magnitude of G assumes less significance as M increases into the supersonic range, because the reduced frequency at flutter generally decreases as supersonic Mach number increases, so that $C(k)$ approaches $1 + iO$. The objective was to try a minimum modification of the existing method.

As a further consequence of this objective, no compressibility modification has been incorporated for the noncirculatory terms of Eq. (3). The noncirculatory forces are associated with the pumping action of the wing during oscillation and hence are functions of the oscillatory velocity component perpendicular to the wing surface and not of freestream velocity directly as is the case for the circulatory terms.[1, 3, 4] The noncirculatory contribution to the section aerodynamic loading thus decreases as reduced frequency decreases, and in most flutter analyses appears to be small though not negligible. Since the use of steady-flow aerodynamic parameters in the present method probably makes it suitable only for cases involving low-to-moderate reduced frequencies, the velocity component perpendicular to the wing surface will, in general, be small. Therefore, the effect of compressibility on the noncirculatory flow associated with this small oscillatory normal velocity component is ignored.

The complete expressions thus obtained for the oscillatory section lift and pitching moment on a section of a swept wing are[4]

$$
\begin{aligned}
P = -\pi\rho b^2[\ddot{h} + V_n\dot{\theta} + V_n\dot{\sigma}\tan\Lambda_{ea} - {} \\
ba(\ddot{\theta} + V_n\dot{\tau}\tan\Lambda_{ea})] - c_{\ell_{\alpha,n}}\rho V_n b C w
\end{aligned}
\tag{4}
$$

and

$$
\begin{aligned}
M_\alpha = {} & -\pi\rho b^4(\tfrac{1}{8} + a^2)(\ddot{\theta} + V_n\dot{\tau}\tan\Lambda_{ea}) + \pi\rho b^2 V_n(\dot{h} + {} \\
& V_n\sigma\tan\Lambda_{ea}) + \pi\rho b^3 a(\ddot{h} + V_n\dot{\sigma}\tan\Lambda_{ea}) + \pi\rho b^2 V_n^2(\theta - {} \\
& ab\tau\tan\Lambda_{ea}) - 2\pi\rho V_n b^2[\tfrac{1}{2} - (a - a_{c,n})C c_{\ell_{\alpha,n}}/2\pi]w
\end{aligned}
\tag{5}
$$

where the downwash expression w is given by

$$w = \dot{h} + V_n\theta + V_n\sigma \tan\Lambda_{ea} + b(c_{\ell_{\alpha,n}}/2\pi + a_{c,n} - a)(\dot{\theta} + V_n\tau \tan \Lambda_{ea}) \quad (6)$$

Equations for flutter analysis employing these expressions are given in Ref. 4.

To review briefly, if aerodynamic parameters for two-dimensional incompressible flow are employed, the modified strip analysis reduces to the method of Barmby, Cunningham, and Garrick.[3] In addition, if the elastic axis is unswept, both methods become identical with that of Theodorsen.[1] On the other hand, if the flow is steady, the modified strip analysis reduces to the conventional steady-state form [Eq. (1)], which itself is employed in the simplified steady-state, flutter-analysis method of Pines.[8,13–16]

The principles of the modified strip analysis may be summarized as follows: Variable section lift-curve slope and aerodynamic center are substituted, respectively, for the two-dimensional incompressible-flow values of 2π and quarter chord which were employed by Barmby, Cunningham, and Garrick.[3] Spanwise distributions of these steady-flow section aerodynamic parameters, which are pertinent to the desired planform and Mach number, are used. Appropriate values of Mach number-dependent circulation functions are obtained from two-dimensional unsteady compressible-flow theory.

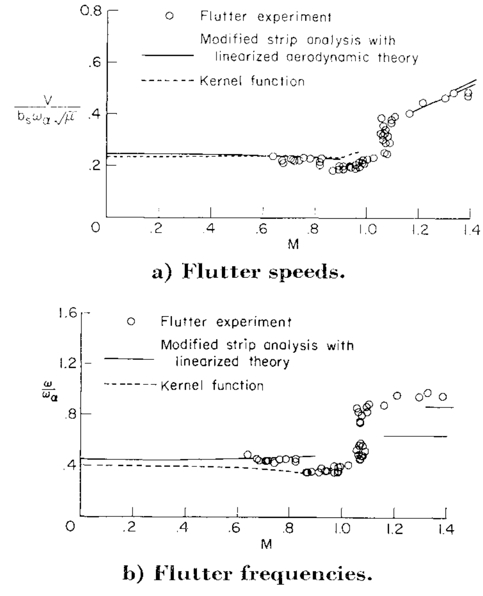
Use of the modified strip analysis avoids the necessity of re-evaluating a number of loading parameters for each value of reduced frequency, since only the modified circulation functions, and of course the reduced frequency itself, vary with frequency. It is therefore practical to include in the digital computing program a very brief logical subroutine, which automatically selects reduced-frequency values that converge on a flutter solution. The problem of guessing suitable reduced-frequency values is thus eliminated, so that a large number of flutter points can be completely determined in a single brief run on the computing machine. If necessary, it is also practical to perform the calculations manually.

Results and Discussion

Numerical Results

Flutter characteristics have been calculated by the modified strip analysis and compared with results of other calculations and with experiments for Mach numbers up to 15.3 and for wings with sweep angles from 0° to 52.5°, aspect ratios from 2.0 to 7.4, taper ratios from 0.2 to 1.0, and center-of-gravity positions between 34% chord and 59% chord.[4–11] These ranges probably cover the great majority of wings that are of practical interest with the exception of very low-aspect-ratio surfaces such as delta wings and missile fins.

Figure 1 shows some initial bending-torsion flutter data measured in the Langley transonic dynamics tunnel in air and in Freon-12 for a cantilevered 45° swept wing with taper ratio 0.6 and aspect ratio 4.[10] The open circular symbols indicate flutter points measured in air, and the open square symbols indicate data measured in Freon-12. The results of an attempt to duplicate this set of data by a three-mode modified strip analysis are indicated by the corresponding solid symbols. Flutter conditions calculated by use of steady-flow aerodynamic parameters obtained from linearized lifting-surface theory are represented by plain symbols, and values calculated by use of experimentally determined steady-state pressure distributions are indicated by flagged symbols. Note first that the flutter data measured in air are duplicated quite satisfactorily in both subsonic and transonic ranges. In particular, the transonic minimum flutter speed is accurately predicted (Fig. 1a). Next observe that the levels of flutter-speed index obtained with Freon-12 in the transonic range are about 70% higher than corresponding values for air. This



a) Flutter speeds.

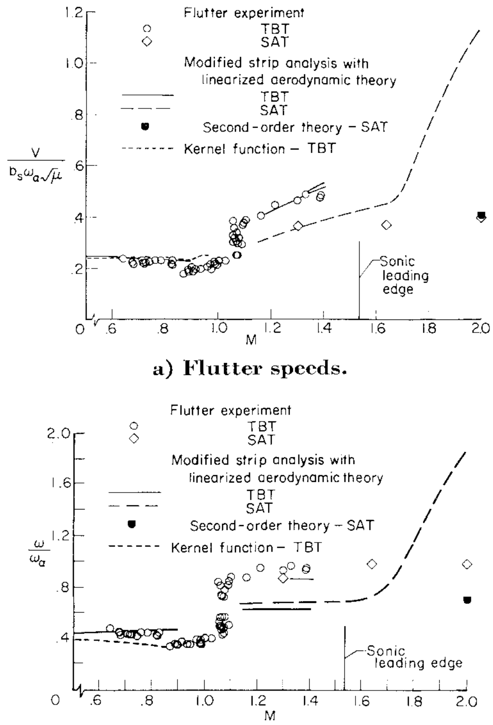
b) Flutter frequencies.

Fig. 2 Flutter characteristics of highly tapered swept wing: $\Lambda_{c/4} = 45°$, $\lambda = 0.2$, and $A = 4.0$.

difference is also predicted by the modified strip analysis and is associated with the large differences in mass ratio between tests in air and in Freon-12. Mass ratios for the tests in Freon-12 were of the order of 8 to 12, whereas mass ratios for the transonic tests in air were as high as 260. Agreement between calculated and measured flutter frequencies is also satisfactory (Fig. 1b).[10]

Figure 2 shows flutter data obtained in the Langley transonic blowdown tunnel for a cantilevered 45° swept wing with taper ratio 0.2 and aspect ratio 4.[6] In the subsonic range, the four-mode modified strip analysis is in good agreement with results of the kernel-function method[17] and with experiment. Note the abrupt rise of flutter speed near Mach number 1.07 (Fig. 2a). This rise was accompanied by a corresponding increase of flutter frequency (Fig. 2b) and thus indicated an abrupt change of flutter mode. In the low supersonic range, the modified strip analysis indicated two flutter solutions having essentially the same flutter speed but different flutter frequencies. One of these calculated frequencies was near the level of experimental subsonic flutter frequency, whereas the other was higher and close to supersonic measured flutter frequencies. Thus, the modified strip analysis also predicted a sudden change in flutter mode.

Figures 3a and 3b show these same results from the transonic blowdown tunnel (TBT) in comparison with flutter data for the same wing measured in the Langley supersonic aeroelasticity tunnel (SAT).[6] In the low supersonic range, the apparent discrepancy in flutter speed levels obtained in the two tunnels (Fig. 3a) is also predicted by the modified strip analysis and is caused primarily by differences in mass ratio for the two sets of tests. This figure also shows one of the difficulties that may be encountered when linearized aerodynamic theory is used in the modified strip analysis. Calculated flutter speeds relating to the data from the supersonic aeroelasticity tunnel turn sharply upward and become highly unconservative after the wing leading edge becomes supersonic and the local aerodynamic centers move rearward to the vicinity of the local centers of gravity. When the local aerodynamic centers lie close to the local centers of gravity, the flutter speeds calculated by the modified strip analysis become quite sensitive to small changes in the aerodynamic-center positions. Under these circumstances, which generally occur in the supersonic range, the combination of linearized aerodynamic theory and the modified strip analysis may yield highly unconservative flutter results, because, as in the present case,



a) Flutter speeds.

b) Flutter frequencies.

Fig. 3 Flutter characteristics of highly tapered swept wing: $\Lambda_{c/4} = 45°$, $\lambda = 0.2$, and $A = 4.0$.

linearized supersonic aerodynamic theory characteristically predicts aerodynamic centers that are too far rearward. It is apparent that under such circumstances adequate flutter prediction will require aerodynamic parameters to be determined by methods more accurate than linearized aerodynamic theory, e.g., from nonlinear aerodynamic theories that include the effects of finite wing thickness, or from measured steady-state loads. In Fig. 3a, use of aerodynamic parameters obtained from the Busemann second-order theory yielded quite accurate flutter speed at $M = 2.0$. Further use of aerodynamic parameters calculated from nonlinear theory is illustrated in Ref. 7.

Figure 4 contains flutter data for a rectangular wing of aspect ratio 2.[8,18] This wing was not cantilever-mounted, as were the previously mentioned wings. Instead, the root of this rectangular wing was attached to a flexible arm, which permitted the wing panel to flap and pitch almost as a rigid body. Consequently, the two natural vibration modes employed in the analysis were predominantly flapping and pitching, although some bending and torsion were also present. Although the experimental data do not extend into the subsonic range, the subsonic flutter speeds and frequencies given by the strip method appear reasonable in comparison with the transonic data. At Mach numbers above 1.0, the strip calculations[8] compare favorably with experiments and with calculations by the quasi-steady, second-order theory of Van Dyke,[19,20] which is closely comparable to piston theory and includes the effects of finite wing thickness. Agreement with experiment is good up to hypersonic speeds. In the modified strip analysis, the compressibility modification employed in the circulation function becomes zero as the component of Mach number normal to the leading edge approaches 1.0. Hence the modified strip analysis is not suitable for application to unswept wings in the near-sonic range unless measured aerodynamic parameters or empirical modifications are employed in the computation of the circulation functions. However, the simplified steady-state method of Pines,[13] which is the zero-frequency limiting case of the modified strip analysis, may be used transonically for unswept wings.

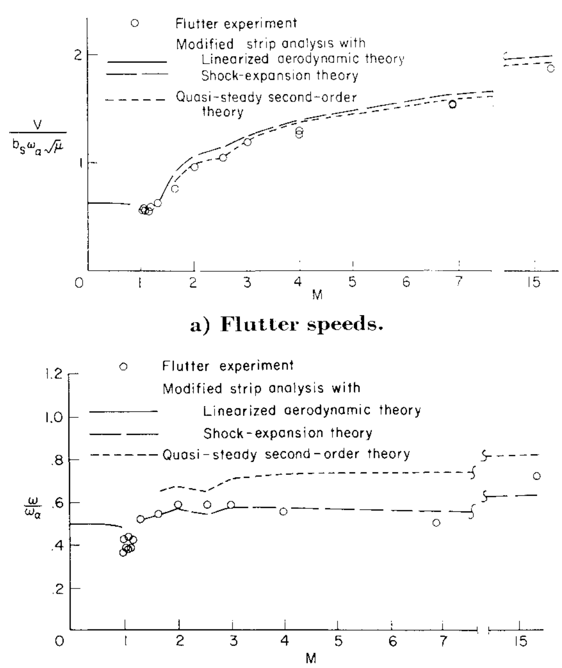
Limitations of the Modified Strip Analysis

The modified strip analysis is, of course, subject to the usual planform limitations that apply to strip methods in general; that is, it is not considered suitable for application to wings of low aspect ratio. Satisfactory results have been shown herein for an unswept wing with a panel aspect ratio of 1.0.[8] Good results have also been obtained for a 45° swept wing with a panel aspect ratio of 0.9.[4] These aspect ratios, however, are probably close to the lower limit for which the modified strip analysis or any strip method would be suitable. In addition, the modified strip analysis as presently formulated does not permit camber deformations of wing sections normal to the elastic axis, although a camber mode could be introduced without great difficulty. Note that even with the present formulation, however, camber of streamwise sections may be represented for swept wings.

Use of aerodynamic parameters associated with steady-flow conditions implies that suitability of the method for high reduced frequencies may be questionable. However, good flutter results have been obtained with reduced frequencies as high as 0.3.[6] None of the numerous flutter calculations, which have been made by the modified strip analysis, have been limited by this condition.

Because of the nature of the compressibility modification applied to the circulation function, the modified strip analysis is not applicable when the component of Mach number normal to the wing leading edge is near 1.0. For swept wings this condition occurs in the supersonic range and does not constitute a serious limitation, because calculated flutter boundaries generally vary smoothly with Mach number in this region. Curves may therefore be faired through calculated flutter characteristics with confidence. For unswept wings, however, this limitation precludes application of the modified strip analysis in the transonic range unless measured aerodynamic parameters or empirical modifications are employed in the computation of the circulation functions. Nevertheless, the zero-frequency limiting case of the modified strip analysis may be employed transonically for these wings.

Finally, in cases for which the local aerodynamic centers lie close to the local centers of gravity, the aerodynamic parameters employed in the modified strip analysis should be determined by the most accurate means available, since in



a) Flutter speeds.

b) Flutter frequencies.

Fig. 4 Flutter characteristics of rectangular wing: $\Lambda_{c/4} = 0$, $\lambda = 1.0$, and $A = 2.0$.

these cases the calculated flutter speeds are quite sensitive to small changes in the locations of the local aerodynamic centers. This limitation does not seem to be peculiar to the modified strip analysis, but appears to apply to other flutter-analysis methods as well.[8]

Conclusions

A modified strip analysis has been developed for rapidly predicting flutter of finite-span, swept or unswept wings at subsonic to hypersonic speeds. The method employs distributions of aerodynamic parameters which may be evaluated from any suitable linear or nonlinear steady-flow theory or from measured steady-flow load distributions for the undeformed wing. The method has been shown to give good flutter results for a broad range of wings at Mach number from 0 to as high as 15.3.

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